Linear Equations in Two Variables

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Introduction

- In earlier classes, you have studied linear equations in one variable. Example of a linear equation in one variable is $2x+5=0$. You know that such equations have a unique (i.e. one and only one) solution. You may also remember how to represent the solution of a linear equation in one variable on an number line.
- In this chapter, the knowledge of linear equations in one variable shall be recalled and extended to that of two variables.
- You will learn about the solution of linear equations in two variables.
- Further, you will learn about the geometric representation of linear equations in two variables.
- Further, you will learn about the equations of lines parallel to x-axis and y-axis.

Linear Equations

- **Linear equation** is an equation which makes a straight line when graphed.
- **Linear equation in two variables** means a linear equation which consists of two different variables. For example, $2x+3y=0$ is a linear equation in two variables.
- **While solving an equation**, you must always keep the following points in mind:

  The solution of a linear equation is not affected when:
  - the same number is added to (or subtracted from) both sides of the equation.
  - you multiply or divide both the sides of the equation by the same non-zero number.
- **The standard form of a linear equation in two variables** is of the form **$ax+by+c=0$**, where a, b and c are real numbers, and a and b are not both zero, is called a linear equation in two variables. For example, $7x+6y+5=0$ is of the form $ax+by+c=0$ where $a=7$, $b=6$ and $c=5$.
- Equations of the type **$ax+b=0$** are also examples of linear equations in two variables because they can be expressed as **$ax+0.y+b=0$**. For example, $4x-3=0$ can be written as $4x+0.y-3=0$

Solution of a Linear Equation

- A linear equation in two variables has **infinitely many solutions**.
- Let us try to find the solutions of the equation $x+2y=6$.

One of the way to find two solutions of an equation is put value of x as zero and then put value of y as zero.

If we put $x=0$, then the equation becomes $0+2y=6$, or $y=3$. Thus, $x=0$ and $y=3$ is a solution of the given equation.

Similarly, if we put $y=0$, then the equation becomes $x+2(0)=6$, or $x+0=6$, or $x=6$. Thus, $x=6$ and $y=0$ is another solution of the given equation.

Now, we can find even more solutions of the given equation, by taking the value of x as 1, 2, 3 etc.

So, we list down the solutions of the equation $x+2y=6$ in the following table:

x	0	6	1	2	3
y	3	0	2.5	2	1.5

Graph of a Linear Equation in Two Variables

- **Graph of a linear equation** in two variables is the geometric representation of the solutions of the linear equation in two variables.
- **Graph of linear equation in two variables** can be drawn by plotting the solutions of the linear equation on a graph paper and joining those points to form a line.
- For example, let us draw the graph of the equation $x+2y=6$.

  The solutions of the above equation can be obtained by assuming some values of x and finding corresponding values of y for each value of x. Some solutions can be expressed in the form of a table as given below:

x	0	6	2	4	…
y	3	0	2	1	…

Graph of a Linear Equation in Two Variables (Contd..)

- Let us plot the points (0,3), (6,0), (2,2) and (4,1) on a graph paper. Now, join any two of these points and obtain a line. Let us call this line as AB (see Fig. 1.1).
- Note that:
  - Every point whose coordinates satisfy the given equation lies on the line AB.
  - Every point (a, b) on the line AB gives a solution x=a, y=b of the given equation.
  - Any point, which does not lie on the line AB, is not a solution of the given equation.

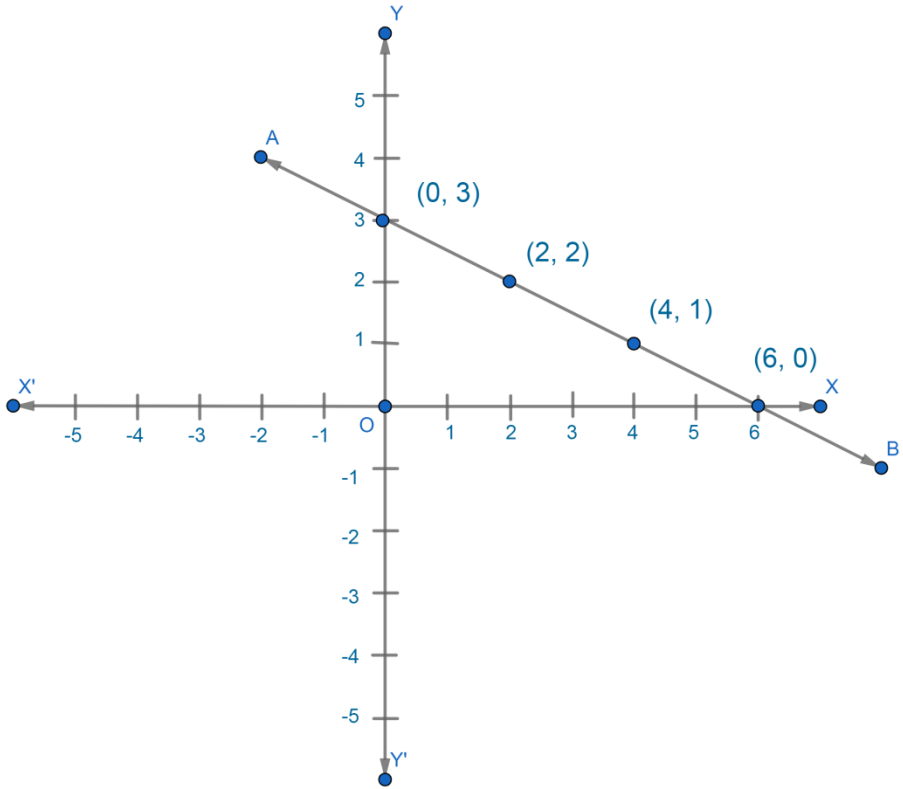

Fig. 1.1

Equations of Lines Parallel to the x-axis and y-axis

- The graph of the linear equation x=a is a straight line parallel to y-axis.
- Let us take an example. Consider the equation x=2. Now, this equation can be expressed as x+0.y=2. This has infinitely many solutions. Some of the solutions of this equation are listed in the below table:

x	2	2	2	2	…
y	0	1	2	3	…

Now, let us plot the solutions of this equation on a graph paper (see Fig. 1.2). We observe that the graph of the equation x=2 is a straight line parallel to y-axis.

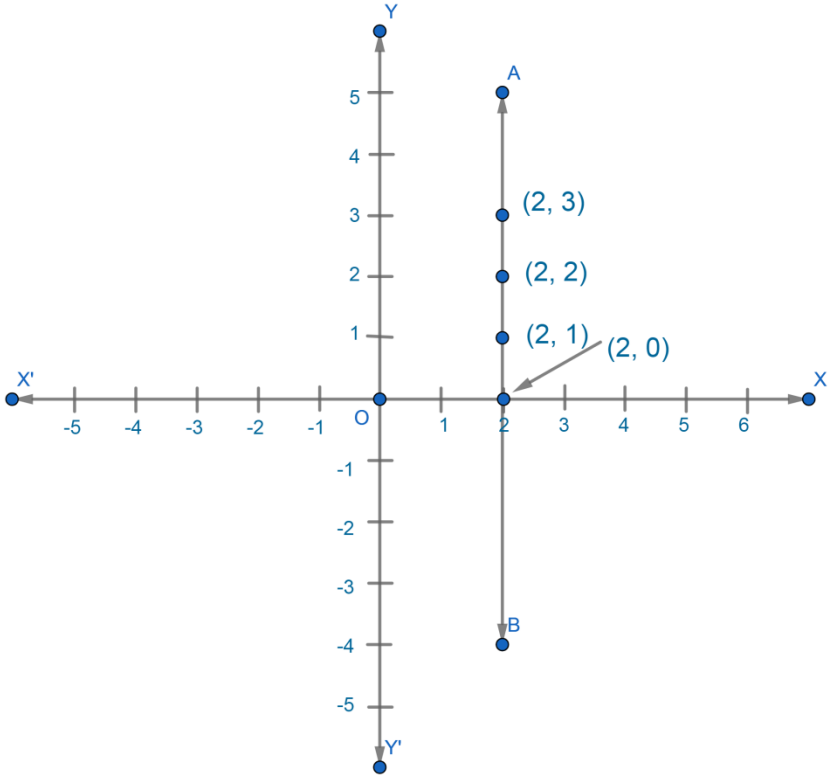

Fig. 1.2

Equations of Lines Parallel to the x-axis and y-axis (Contd..)

- The graph of the linear equation y=a is a straight line parallel to x-axis.
- Let us take an example. Consider the equation y=3. Now, this equation can be expressed as 0.x+y=3. This has infinitely many solutions. Some of the solutions of this equation are listed in the below table:

x	0	1	2	3	…
y	3	3	3	3	…

Now, let us plot the solutions of this equation on a graph paper (see Fig. 1.3). We observe that the graph of the equation y=3 is a straight line parallel to x-axis.

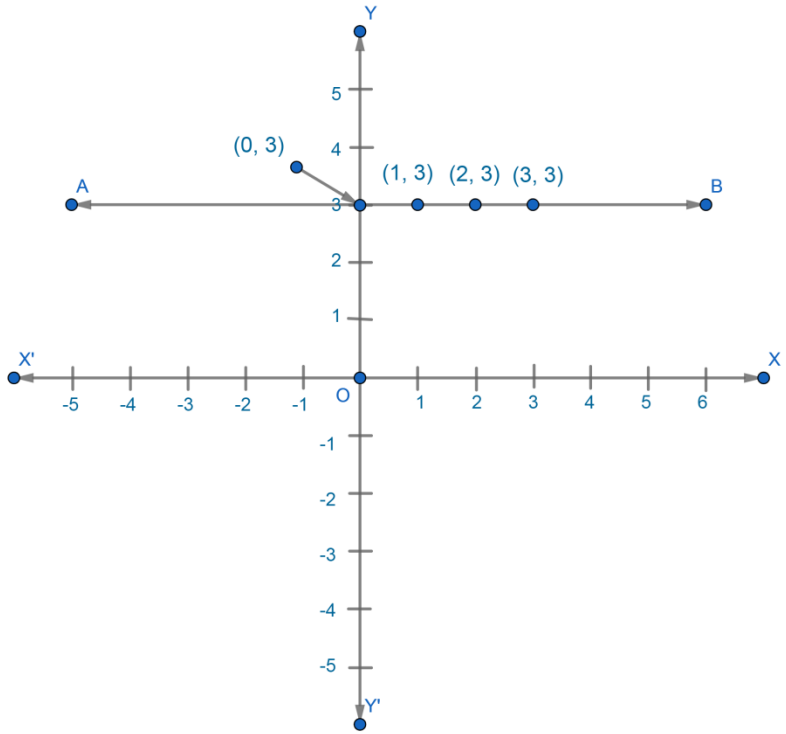

Fig. 1.3

Summary

- An equation of the form $ax+by+c=0$, where a, b and c are real numbers, such that a and b are not both zero, is called a linear equation in two variables.
- A linear equation in two variables has infinitely many solutions.
- The graph of every linear equation in two variables is a straight line.
- $x=0$ is the equation of the y-axis and $y=0$ is the equation of the x-axis.
- The graph of $x=a$ is a straight line parallel to the y-axis.
- The graph of $y=a$ is a straight line parallel to the x-axis.
- An equation of the type $y=mx$ represents a line passing through the origin.
- Every point on the graph of a linear equation in two variables is a solution of the linear equation. Moreover, every solution of the linear equation is a point on the graph of the linear equation.

THANK YOU